

# The parameter space for tree-level hybrid inflation

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## Abstract

In a large region of parameter space, the tree-level hybrid inflation model is likely to be invalidated by loop corrections. In particular, this is likely to be the case if both quartic couplings are of order unity, as is often supposed. It is likely also to be the case if there is an ultra-violet cutoff far below the Planck scale, ( $\Lambda_{UV} \lesssim 10^9 \text{ GeV} (V_0^{1/4}/1 \text{ MeV})^{2/5}$ , where  $V_0$  is the height of the potential) unless one allows field values bigger than the cutoff.

1. The original tree-level hybrid inflation model [1] involves the mass of the inflaton field  $\phi$ , the coupling  $\lambda' \phi^2 \psi^2$  of the inflaton to another field  $\psi$ , and the self-coupling  $\lambda \psi^4$ . It was immediately noted that the model can give a density perturbation of the required magnitude with the natural choice  $\lambda \sim \lambda' \sim 1$ . This feature, desirable in itself, yields the bonus [2] that all relevant field values are far below the Planck scale, making it feasible to justify the neglect of non-renormalizable terms in the potential [3, 4].

Supersymmetric realizations of the tree-level model were soon proposed [2, 3], which seemed to confirm its theoretical viability. Unfortunately, with supersymmetry in place it became sensible to evaluate the 1-loop correction to the potential. In both models, the loop corrections was found [5, 6] to be large, necessarily dominating the tree-level term if both couplings are of order 1.<sup>1</sup>

In this note I argue that the loop correction in a generic model will be at least as big as the one found in these particular cases, unless there are unforeseen cancellations between unrelated parameters. On this basis, I describe the maximal region of parameter space in which the tree-level model can be valid. As already indicated, it does not include the regime of unsuppressed couplings, which unfortunately is still in common use.<sup>2</sup> I shall

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<sup>1</sup>One of the models had the  $F$  term dominating [2, 3] while the other had the  $D$  term dominating [3]. The models actually made the potential completely flat during inflation, but supergravity corrections generate [2, 3] a non-zero (in fact generically rather too large) curvature for the potential. The loop correction was actually first evaluated [5, 6] for slight variants of the original models.

<sup>2</sup>The two most recent examples are [7] which invokes this regime to construct a model of inflation with TeV-scale quantum gravity, and [8] which uses it to construct a model of baryogenesis.

quote without comment some well-known results, referring for details to a recent review of inflation [4].

2. The original tree-level hybrid inflation model [1] is defined by

$$V(\phi, \psi) = V_0 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\lambda'\psi^2\phi^2 - \frac{1}{2}|m_\psi^2|\psi^2 + \frac{1}{4}\lambda\psi^4. \quad (1)$$

Inflation takes place in the regime  $\phi^2 > \phi_c^2 \equiv |m_\psi^2|/\lambda'$ , where  $\psi$  is driven to zero and the inflaton potential is

$$V = V_0 + \frac{1}{2}m^2\phi^2. \quad (2)$$

The constant term  $V_0$  is assumed to dominate during inflation.

The last term of Eq. (1) serves only to determine the vacuum expectation value (vev) of  $\psi$ , achieved when  $\phi$  falls below  $\phi_c$ . Using that fact that  $V_0$  vanishes in the vacuum, one learns that the vev is

$$\langle\psi\rangle \equiv M = 2V_0^{1/2}/|m_\psi| = \lambda^{-1/2}|m_\psi|, . \quad (3)$$

If the last term of Eq. (1) is replaced by a higher (but not huge) power of  $\psi$  one still has roughly

$$M \sim 2V_0^{1/2}/|m_\psi|, \quad (4)$$

and for the most part I will invoke only this result, making no explicit use of the parameter  $\lambda$ . The results will therefor not be sensitive to the precise power appearing in the the last term of Eq. (1).

During inflation, it is useful to define parameters

$$\eta \equiv \frac{m^2 M_P^2}{V_0} \quad (5)$$

$$\eta_\psi \equiv \frac{|m_\psi^2| M_P^2}{V_0} \sim \frac{4M_P^2}{M^2}. \quad (6)$$

As usual,  $M_P \equiv (8\pi G)^{-1/2} = 2.4 \times 10^{18}$  GeV is the Planck scale. Slow-roll inflation requires  $\eta \ll 1$ , and a prompt end to inflation at  $\phi_c$  requires  $\eta_\psi \gg 1$ . The latter condition can be interpreted loosely [9], but the former has to be tight because the spectral index  $n$  is predicted as  $n - 1 = 2\eta$  whereas observation requires  $|n - 1| \ll 1$ . A preliminary estimate from recent observations is [10]  $|n - 1| < 0.05$ , so it seems reasonable to assume

$$\eta \lesssim 0.025 \quad (7)$$

$$\eta_\psi \gtrsim 1 \quad (8)$$

The scales probed by COBE leave the horizon  $N < 60$   $e$ -folds before the end of slow-roll inflation, when the field is

$$\phi^2 = e^{2\eta N} \phi_c^2 \simeq \phi_c^2 \equiv |m_\psi^2|/\lambda'. \quad (9)$$

(The approximate equality corresponds to  $e^{2\eta N} \sim 1$  which is good enough for order-of-magnitude estimates since  $\eta < 0.025$  requires  $e^{2\eta N} \lesssim 5$ .) The COBE normalization of the spectrum of the density perturbation gives the constraint [1, 2, 4]

$$\lambda' = 2.8 \times 10^{-7} e^{2\eta N} \eta^2 \eta_\psi \quad (10)$$

$$\simeq 3 \times 10^{-7} \eta^2 \eta_\psi. \quad (11)$$

After imposing this constraint, the requirement that  $V_0$  dominates the inflaton potential becomes

$$\frac{V_0}{M_{\text{P}}^4} \ll 3 \times 10^{-7} \eta. \quad (12)$$

3. The 1-loop correction to the inflationary potential coming from  $\psi$  is

$$\Delta V = \frac{1}{32\pi^2} m_\psi^4(\phi) \ln \left( \frac{m_\psi(\phi)}{Q} \right) \quad (13)$$

where

$$m_\psi^2(\phi) \equiv (\lambda' \phi^2 - |m_\psi^2|) = \lambda'(\phi^2 - \phi_c^2). \quad (14)$$

(A constant of order 1 has been dropped in the argument of the log, which depends on the renormalization scheme and does not affect the conclusions.) In this expression,  $Q$  is the renormalization scale at which the parameters of the tree-level potential should be evaluated. Its choice is arbitrary, and if all loop corrections were included the total potential would be independent of  $Q$ . In any application of quantum field theory, one should choose  $Q$  so that the total 1-loop correction is small, hopefully justifying the neglect of multi-loop correction. In the present context this is achieved by choosing

$$Q \sim \sqrt{\lambda'} \phi_c. \quad (15)$$

In order to justify the omission of a quartic term  $\propto \phi^4$  in the tree-level potential, and also to protect the parameters against radiative corrections, one assumes that the underlying theory is supersymmetric. The above loop correction will then be accompanied by the loop corrections coming from the spin-zero and spin-half superpartners of  $\psi$ . If supersymmetry were unbroken, the total loop correction would vanish, but supersymmetry is necessarily broken during inflation. (The scale of susy breaking, defined essentially as the magnitude of the auxiliary field(s) breaking susy, cannot be less than  $V^{1/2}$ .) The breaking can be either spontaneous or soft, but in either case the superpartners couple to  $\phi$  with the *same* coupling strength  $\lambda'$ . The total loop correction from  $\psi$  and its superpartners is<sup>3</sup>

$$\Delta V = \frac{1}{32\pi^2} \left( m_\psi^4(\phi) \ln \left( \frac{m_\psi(\phi)}{Q} \right) + m_{\tilde{\psi}}^4(\phi) \ln \left( \frac{m_{\tilde{\psi}}(\phi)}{Q} \right) - 2m_{\text{f}}^4(\phi) \ln \left( \frac{m_{\text{f}}(\phi)}{Q} \right) \right), \quad (16)$$

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<sup>3</sup>The mass matrix is taken to be diagonal for simplicity, since the non-diagonal case does not seem to introduce any essentially new feature.

where

$$m_{\tilde{\psi}}^2(\phi) = \lambda' \phi^2 + m_{\tilde{\psi}}^2 \quad (17)$$

$$m_f(\phi) = \sqrt{\lambda'} \phi + m_f. \quad (18)$$

Here  $\tilde{\psi}$  is the field of the scalar partner, while  $f$  denotes the spin-half partner. The mass  $m_f \equiv m_f(0)$  vanishes if the full potential respects the symmetry  $\phi \rightarrow -\phi$  of the original, which we assume.

The mass-squared  $m_{\tilde{\psi}}^2$  may be either positive or negative. In the former case,  $\tilde{\psi}$  vanishes both during inflation and in the true vacuum, automatically justifying its omission in the tree-level potential. In the latter case, the omission of  $\tilde{\psi}$  is strictly justified only if  $|m_{\tilde{\psi}}| = |m_{\psi}|$ , the only effect of including it then being the trivial replacement  $\psi^2 \rightarrow \psi^2 + \tilde{\psi}^2$ . However, its omission is justified in practice also if  $|m_{\tilde{\psi}}|$  is *roughly* of order  $|m_{\psi}|$ . Indeed, if  $|m_{\tilde{\psi}}| < |m_{\psi}|$ , the field  $\tilde{\psi}$  is held at the origin until the field  $\psi$  is destabilized, making the former ineffective except for a modest increase in the value of  $V_0$ . The opposite case may be discounted because it is equivalent to an interchange of the labels  $\psi$  and  $\psi'$ . In the following we therefore assume

$$|m_{\tilde{\psi}}| < |m_{\psi}|. \quad (19)$$

I am going to argue that with these assumptions, the derivatives of Eq. (16) will be at least of the following order

$$\Delta V' \sim \frac{|m_{\psi}|^4}{\phi_c} = \sqrt{\lambda'} |m_{\psi}|^3 \quad (20)$$

$$\Delta V'' \sim \frac{|m_{\psi}|^4}{\phi_c^2} = \lambda' |m_{\psi}|^2. \quad (21)$$

Consider first the case that  $\eta$  is not too small, so that  $e^{2\eta N}$  is significantly bigger than 1. In this case,  $\lambda' \phi^2$  is significantly bigger than  $|m_{\tilde{\psi}}^2|$  (equivalently,  $\phi$  is significantly bigger than  $\phi_c$ ) while COBE scales leave the horizon, and remains so until the last few  $e$ -folds of inflation. As a result, the  $\psi$  contribution Eq. (13) is given at least roughly by

$$\Delta V \sim \frac{1}{32\pi^2} \left( \lambda'^2 \phi^4 - 2\lambda' |m_{\tilde{\psi}}^2| \phi^2 + |m_{\psi}|^4 \right) \ln(\sqrt{\lambda'} \phi / Q). \quad (22)$$

The  $\phi^4$  term in front of the log disappears when the contributions of the superpartners are included (Eq. (16) with the arguments of the logs identical). In models with spontaneously broken global supersymmetry the  $\phi^2$  term will also typically be cancelled (in other words one will typically have  $m_{\tilde{\psi}}^2 = |m_{\psi}|^2$ ). The coefficient of the constant term will however *not* be cancelled, and keeping only it gives the estimates Eqs. (20) and (21).<sup>4</sup>

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<sup>4</sup>In models with softly broken supersymmetry the  $\phi^2$  term will also survive and in general be dominant, but the contribution of the constant term is still there.

If  $\eta$  is many orders of magnitude below 1, so that  $\phi$  is very close to  $\phi_c$ , Eq. (22) will not be a good approximation for the  $\psi$  contribution. As a result the spin-half contribution to Eq. (16) is now uncanceled (to be precise, half of it remains uncanceled while the other half might be cancelled by the  $\tilde{\psi}$  contribution). Evaluating the derivatives of this contribution again leads to estimates of order Eqs. (20) and (21).

This justifies Eqs. (20) and (21) as rough estimates of the 1-loop contribution of  $\psi$  and its superpartners, to the derivatives of the potential. As there is no reason why this contribution should be cancelled by additional contributions (coming from additional Yukawa couplings or from gauge fields [11]) Eqs. (20) and (21) are also rough lower bounds on the total 1-loop contribution.

For the tree-level model to be valid, the first two derivatives of the loop correction ought to be negligible,  $\Delta V' \ll m^2 \phi_c$  and  $\Delta V'' \ll m^2$ . The second constraint is the strongest, and it may be written

$$\frac{1}{32\pi^2} \lambda' \ll \frac{\eta}{\eta_\psi}. \quad (23)$$

Using the COBE normalization we can write this constraint in various ways. Eliminating  $\eta_\psi$  it becomes

$$\lambda' \ll (\eta/22)^{3/2} \lesssim 4 \times 10^{-5}. \quad (24)$$

We see that  $\lambda'$  has to be quite small. Eliminating instead  $\eta$  we find a result that can usefully be written in terms of the vev  $M$  of  $\psi$ ,

$$\lambda' \ll (42M/M_P)^6. \quad (25)$$

This is a stronger bound if  $M/M_P \lesssim 0.005(\eta/0.025)^{1/4}$ . The third simple possibility is to eliminate  $\lambda'$ , leading to a result that can be written in similar form,

$$\eta \ll (90M/M_P)^4. \quad (26)$$

This is more restrictive than Eq. (7) if  $M/M_P \lesssim 5 \times 10^{-3}$ . Using Eq. (12) we learn that  $V_0^{1/4} \ll 1.6M$ .

We can also use Eq. (25) to obtain a lower bound on the value Eq. (9) of  $\phi_c$ , which can be written

$$\left(\frac{M}{M_P}\right)^4 \left(\frac{\phi_c}{M_P}\right) \gtrsim \left(\frac{V_0^{1/4}}{156M_P}\right)^2 \gtrsim \left(\frac{10^9 \text{ GeV}}{M_P}\right)^5. \quad (27)$$

To obtain the last inequality we set  $V_0^{1/4} \sim 1 \text{ MeV}$ , the smallest possible value since reheating must occur before nucleosynthesis.

4. The tree-level potential Eq. (1) ignores nonrenormalizable terms. They are of the form  $\lambda_{mn} \Lambda_{UV}^{4-m-n} \phi^m \psi^n$ , with  $m+n \geq 5$ , where  $\Lambda_{UV}$  is the ultra-violet cutoff for the effective field theory relevant during inflation. These terms summarize the physics which is ignored by the effective field theory.

Since quantum gravity certainly become significant on Planckian scales one must have  $\Lambda_{\text{UV}} \lesssim M_{\text{P}}$ , but  $\Lambda_{\text{UV}}$  will be smaller if the effective field theory breaks before Planckian scales are reached. This could happen in three ways. First, a different field theory may take over, containing fields that have been integrated out in the effective theory. Second, field theory may give way to string theory. Third, the scale of quantum gravity could be lower than  $M_{\text{P}}$  because there are extra dimensions with a large compactification radius.

The coefficients  $\lambda_{mn}$  are generically of order 1, so their neglect is normally justified only if all field values are  $\lesssim \Lambda_{\text{UV}}$ .<sup>5</sup> Eq. (27) implies that this is possible only if

$$\Lambda_{\text{UV}} \gtrsim 10^9 \text{ GeV} \left( V_0^{1/4} / 1 \text{ MeV} \right)^{2/5}. \quad (28)$$

5. We noted already that our results remain valid if the last term of Eq. (1) is replaced by a non-renormalizable term. Let us ask what happens if the term  $\frac{1}{2} \lambda' \phi^2 \psi^2$  is replaced by a non-renormalizable term, specifically the term  $\phi^4 \psi^2 / \Lambda_{\text{UV}}^2$  proposed by Randall et al. [9]. This gives  $\phi_c^2 = \Lambda_{\text{UV}} |m_\psi|$  instead  $\phi_c^2 = |m_\psi^2| / \lambda'$ . Eqs. (26) and (27) are unaffected, while Eqs. (24) and (25) become

$$\frac{V_0^{1/2}}{2M\Lambda_{\text{UV}}} \lesssim \left( \frac{42M}{M_{\text{P}}} \right)^6 \quad (29)$$

$$\frac{V_0^{1/2}}{2M\Lambda_{\text{UV}}} \lesssim \left( \frac{\eta}{22} \right)^{3/2} \lesssim 4 \times 10^{-5}. \quad (30)$$

Invoking again<sup>6</sup> the condition  $M \lesssim \Lambda_{\text{UV}}$ , these inequalities become lower bounds on  $\Lambda_{\text{UV}}$ . Invoking again  $V_0^{1/4} \gtrsim 1 \text{ MeV}$ , the first inequality becomes  $\Lambda_{\text{UV}} \gtrsim 10^{12} \text{ GeV}$ , significantly stronger than Eq. (28).

6. In summary, the parameter space for tree-level hybrid inflation is strongly constrained. It does not include the regime where both dimensionless couplings are unsuppressed. Nor does it include the regime where all relevant field values are far below the Planck scale. The second result is perhaps the most interesting, because it means that tree-level hybrid inflation is unviable if there is an ultra-violet cutoff far below the Planck scale, unless one is willing to allow field values much bigger than the cutoff.

## Acknowledgements

I am indebted to Toni Riotto for many useful discussions.

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<sup>5</sup>In the context of global supersymmetry with minimal kinetic terms, the holomorphy of the superpotential allows one to eliminate some of the non-renormalizable terms by imposing suitable internal symmetries but they are still dangerous in the full supergravity theory [4].

<sup>6</sup>In advocating this condition we differ from Randall et al. [9], who advocate  $M \sim M_{\text{P}}$  even in the presence of the ultra-violet cutoff.

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